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On planning production and distribution with disrupted supply chains

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Abstract. This paper presents a model for short-term time-horizon production and distribution planning of a manufacturing company located in the middle of a supply chain. The model focuses on an unbalanced market with broken supply chains. This reflects the state of the current post-COVID-19 economy, which is additionally struggling with even more uncertainty and disruptions due to the Russian aggression against Ukraine. The manufacturer, operating on the post-pandemic and post-war market, on the one hand observes a soaring demand for its products, and on the other faces uncertainty regarding the availability of components (parts) used in the manufacturing process. The goal of the company is to maximise profits despite the uncertain availability of intermediate products. In the short term, the company cannot simply raise prices, as it is bound by long-term contracts with its business partners. The company also has to maintain a good relationship with its customers, i.e. businesses further in the supply chain, by proportionally dividing its insufficient production and trying to match production planning with the observed demand. The post-COVID-19 production-planning problem has been addressed with a robust mixed integer optimisation model along with a dedicated heuristic, which makes it possible to find approximate solutions in a large-scale real-world setting.

Keywords: production, optimisation techniques, simulation modelling, programming models, transportation economics
JEL: C44, C61, L90

1. Introduction

The COVID-19 pandemic has changed the way markets and economies function across almost all industry branches. Severe disturbances in how supply chains operate can be currently observed all over the world. The just-in-time supply model is no longer feasible for many companies (Brakman et al., 2020). This complicated situation even worsened due to the Russian aggression against Ukraine, which caused damages to Ukraine's economy, the seizure of export routes and further complications to the processes of production and distribution of goods, the latter being predominantly the consequence of global sanctions against the aggressor (Mbah & Wasum, 2022).

Another phenomenon observed across many industry sectors is decreased availability of consumer products such as electronics, computers, or vehicles

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(Chen et al., 2021). The globally-observed changes in the availability of raw materials and semi-products of critical components have changed the way manufacturing companies should operate and intermediate products or final goods be provisioned to companies further in the supply chain, customers and resellers.

Several papers show that the COVID-19 pandemic, and later the war in Ukraine, have drastically influenced the efficiency of global supply chains. Cai and Luo (2020) reviewed the impact of COVID-19 on the manufacturing chain, pointing out its negative influence on raw materials, spare parts, intermediate products and workforce availability. The authors further noticed that on the one hand, companies had to adapt to the chain disturbances, and on the other needed new methods to enhance supply chain resilience. The final conclusion of their study is that in the post-crisis world, the manufacturing supply chain is likely to become regionalised and digitalised.

Singh et al. (2021) pointed out that the COVID-19 pandemic was continuously causing disturbances across all the levels of the economy. It affected the access to crucial resources (employees, logistics, raw materials), essential items (basic food commodities, perishable food items, medicines, diagnostic equipment, personal protection equipment), primary economic sectors (aviation, railway, agriculture, healthcare, FMCG) as well as sectors playing a significant role in modern economies (hospitality, construction, information technology, automotive and textile manufacturing). Similar conclusions could also be found in Queiroz et al. (2020).

In order to mitigate supply chain disturbances, Paul and Chowdhury (2021) considered a theoretical scenario of a manufacturing system where under normal circumstances the production was higher than the demand, but the situation reversed as a result of the COVID-19 pandemic, i.e. the demand surged while the production capacity decreased (supply disturbances). To solve this problem, the authors proposed a model where the reserve storage capacity was expanded and purchases of raw materials from more expensive suppliers were increased with the purpose of speeding up the production process. In their approach, the production plan was represented by a constrained nonlinear optimisation problem which they solved with gradient methods. A simplified version of this model was presented by Shahed et al. (2021), who based it on a profit-maximising manufacturer with a single supplier and a single retailer. This research pointed out the importance of re-implementing inventory management policies in manufacturing companies.

Tsolas and Hasan (2021) proposed economic survivability (understood as a point where a business ceases to be profitable) model to explain decisions of a company operating in a market with high fluctuations of raw materials availability and demand. The authors built a survivability-maximising optimisation model and showed that a company should balance the allocation of its manufacturing plants across multiple regions and ensure diversified supply chain connections between suppliers of raw materials and the factories – even if it leads to a decrease in the overall profit. On the other hand, Li et al. (2020) stressed the role of intelligent manufacturing as a proactive method to mitigate production disruptions caused by a pandemic. They proposed to implement a continuous decision-making model for determining the optimal deployment of resources to strengthen the existing industrial network.

The literature shows that manufacturing and distribution companies are currently facing two types of uncertainties: on the one hand, lack of raw material semi-products and intermediate and key components that affect the production activity, and on the other huge fluctuations in demand coupled with product shortages in many consumer markets that is disruptive to the distribution activities.

For the purpose of this paper, we selected a manufacturing company that experienced a soaring demand for its products and at the same time faced shortages of critical components, which made it impossible for it to operate at full production capacity. The goal of the company was to maximise profits despite uncertain availability of intermediate products. The research presented in the paper focuses on a short-term decision-making horizon. By 'short-term' we mean that the prices agreed upon by the company and its customers were fixed, i.e. the company was bound by long-term pricing contracts. For this reason, and despite limited supply, the product allocation problem could not be simply addressed by raising prices, as the company had to take into consideration long-term relationships with its customers. Moreover, huge fluctuations in the demand were observed on the market and the availability of components critical to the production could not be guaranteed. Businesses struggling with this kind of problems are, for example, automotive dealers, medicine producers and producers of electronic devices.

Our paper presents a novel approach, featuring a short-term model of a market with fixed demand, an insufficient supply of goods and a reduced price flexibility. This approach has been selected in order to analyse the decision-making process in the current post-pandemic economy that is additionally struggling with the effects of the Russian aggression in Ukraine. The manufacturer chosen for the purpose of the study was able to manufacture only a limited number of goods that had to be distributed among companies located further in the supply chain. The goal of the manufacturer was on the one hand to minimise the potential frustration of its customers and on the other to maximise profits.

In order to show our problem in a business setting, let us consider an automotive manufacturer that has long-term contracts and business relations with car dealers. In the short term, the manufacturer cannot adjust the price list. However, since cars yield different profit margins, they can be distributed in many ways among different dealers. The manufacturer cannot ignore that fact that each car dealer made some pre-orders or entered into long-term contracts when the economic situation was different. However, since there generally is a significant shortage of cars, we can observe that several producers e.g. started manufacturing vehicles with different equipment than originally planned (Boston, 2022). For instance, they provide customers with vehicles that have different engine types or are of different colours than it was stated in the original order.

The goal of the paper is to propose an optimisation approach to address the problem of a manufacturer experiencing disruptions of supplies and at the same time soaring demand, all in a short-term decision-making horizon. In order to address the uncertainty of critical component availability, the study adopted the robust optimisation approach of Beyer and Sendhoff (2007). The model assumes that manufacturers of critical components might adopt a similar strategy as the company selected for the purposes of the study, i.e. provide the manufacturer of end products with slightly different components than originally requested.

The paper is constructed in the following way: Introduction is followed by Section 2, where a mathematical model formulation is proposed, Section 3 presents the results of numerical experiments, and Section 4 comprises the conclusions of the study.

2. Problem statement and model

As mentioned before, the profile of the company analysed in this study is one that uses several intermediate components (parts) to manufacture a single product. Companies meeting this criterion include manufactures of computing hardware, cars, e-scooters, electronic appliances and furniture. Our model makes allowances for the fact that manufactures of this kind usually have broad, long-established dealership networks, in the framework of which business relationships have often lasted for a long time and which are an important part of these companies' values. As a consequence of the fact that there are shortages of goods in the market, the customers of such manufacturers are usually willing to accept end products that are slightly different than the original order.

2.1. Managing baseline demand

We are considering a manufacturing company with demand $v_{dn} \in \mathbb{N}_0$, where $d \in D$ is a customer from customer base D, and $n \in N$ is a product from product set N (we use $\mathbb{N}_0$ to denote non-negative integers). The demand of customer $d \in D$ for product $n \in N$ is denoted as v_{dn}, and hence the overall demand is represented by matrix $V = [v_{dn}]$.

In order to manufacture the goods, a set of critical components K is required. The technology matrix is represented by $A = [a_{kn}]$, where $[a_{kn}]$ stands for the number of parts of type k to manufacture good n. $b_k \in \mathbb{N}_0$ is the expected (optimistic) level of the critical component availability. The number of available parts is known only approximately due to disturbances on the market, and so the unknown perturbance is represented by $\xi_k \in \mathbb{N}_0$, $\xi_k \leq \psi_k$ with the maximum perturbance limit of $\psi_k \in \mathbb{N}_0$. Moreover, since there is a possibility of replacing some components with others, we assumed that there is a maximum total perturbation level $\Gamma \in \mathbb{N}_0$, such that $\sum_{k \in K} \xi_k \leq \Gamma$. Hence, for the considered availability of components b, we define the following uncertainty set $\mathbf{U}$ known in the literature (e.g. Li et al., 2011) as the boxed-polyhedral uncertainty:

$$\mathbf{U} = \left\{ \boldsymbol{\xi} \,\middle|\, \mathbf{0} \leq \boldsymbol{\xi} \leq \boldsymbol{\psi} \wedge \left\| \boldsymbol{\xi} \right\|_1 \leq \Gamma \right\}, \tag{1}$$

where we use bold font to represent the vectors of values, i.e. $\boldsymbol{\psi} = [\psi_k]$ and $\boldsymbol{\xi} = [\xi_k]$. Moreover, the $||\,.\,||_1$ notation denotes L^1 norm, i.e. $\left\|\boldsymbol{\xi}\right\|_1 = \sum_{k \in K} \xi_k$.

The studied manufacturer, as already mentioned, operates on a market with significant product shortages, and therefore has to fulfill the demand by offering similar but slightly different products that will be further called 'substitutes'. The product substitution matrix is denoted by $S = [s_{ij}]$, where $s_{ij} = 1$ means that product $i \in N$ can be replaced by product $j \in N$, and 0 means that no replacement is possible. Please note that a product can always be a replacement for itself, and hence the S matrix has 1's on the diagonal (i.e. $s_{ii} = 1$ for $i \in N$). Additionally, we assume that substitutability is a symmetric relation (i.e. $s_{ij} = s_{ji}$), and hence S is symmetric. The company needs to decide how many goods to manufacture in order to satisfy the demand to the fullest possible extent. The production volume is represented by $\mathbf{y} = [y_n]$, $y_n \in \mathbb{N}_0$, and the allocation of those products to customers by $X = [x_{dn}]$, $x_{dn} \in \mathbb{N}_0$. Obviously, the demand can only be fulfilled to the extent allowed by the volume of production ($\sum_{d \in D} x_{dn} \leq y_n$). However, since we are considering a market that is unbalanced in the short term, the demand does not need to be fulfilled with the products that have been actually ordered, as substitutes can be used instead (for example, a green vehicle can be offered to the customer instead of a blue one). Each product $n \in N$ has a unit cost of manufacturing $c_n \geq 0$, and can be sold to customer $d \in D$ at price $p_{dn} \geq c_n$. The goal of the decision-maker, as already stated, is twofold: to maximise the profit and to maintain good long-term relationship with the customer network. For this reason, the business problem is a two-criteria optimisation, but in our approach, the second criterion (the degree to which customers' demand is met) is modelled by a constraint in the optimisation problem. The complete list of symbols used to describe the decision situation is presented in Table 1. Since we are considering manufacturing industries which have both large product portfolios (e.g. offer similar products in different colours or with slightly different technical specifications) and significant number of customers, the decision variables in the model are discrete rather than continuous.

2.2. Optimisation model

As mentioned before, the goal of decision-makers is twofold: besides maximising profits from sales, they strive to maintain good relations with their customer networks (by guaranteeing a minimal level of supply).

This can be presented as the following model:

$$\max \sum_{n\in N, d\in D} x_{dn} p_{dn} - \mathbf{c}^T \mathbf{y}, \tag{2}$$

subject to:

$$A\mathbf{y} \leq \mathbf{b} - \boldsymbol{\xi}, \tag{3a}$$

$$XS \leq VS, \tag{3b}$$

$$X \geq \gamma X^*, \tag{3c}$$

$$\sum_{d\in D} x_{dn} \leq y_n \ \forall n \in N, \tag{3d}$$

$$x_{dn} \in \mathbb{N}_0 \ \forall d \in D, n \in N, \tag{3e}$$

$$y_n \in \mathbb{N}_0 \ \forall n \in N, \tag{3f}$$

where $\boldsymbol{\xi}$ represents the boxed polyhedral uncertainty defined in Equation (1).

Function (2) represents profit maximisation. Typically, each customer $d \in D$ has a long-term relationship with the manufacturer, having negotiated an individual price p_{dn} for a particular product $n \in N$. This means that the price level can vary across the customer base. The profit is denoted as $\rho(X)$. Please note that the increase in the value of y without a corresponding drop in x will always lead to a decrease of the goal Function (2). Therefore, at optimal solution $(X^{opt}, \mathbf{y}^{opt})$, no unsold products will be manufactured, hence the following equation holds true:

$$\rho(X) = \sum_{n \in N, d \in D} x_{dn} p_{dn} - \mathbf{c}^T \mathbf{y} = \sum_{n \in N, d \in D} x_{dn} (p_{dn} - c_n). \tag{4}$$

Equation (3a) assumes that the production should not be greater than the uncertain availability of critical components, where $\mathbf{b}$ is the 'optimistic' availability of components, and $\boldsymbol{\xi}$ is the unknown perturbation. Equation (3b) defines the ranges for product substitution (which involves providing the customers with alternative products to the ones originally requested). Please note that the dimension of XS is $|D| \cdot |N|$ and, hence, in a given matrix row each value controls the total number of products within a group of substitutes for the product corresponding to the column. While Equation (3b) allows the free movement of products across customers, in practice there is still some minimal guaranteed level of matching the actual demand – here presented as γX^* in Equation (3c). Parameter $\gamma \in [0,1]$ represents the substitution rigidity for the minimal required allocation. $\gamma = 0$ means that customers can be freely offered substitutes, while $\gamma = 1$ says that the level of substitute products is minimal. X^* is the maximum feasible solution to the 'pessimistic' version of the problem (i.e. when $\boldsymbol{\xi} = \boldsymbol{\psi}$). One of the possible ways of calculating X^* will be shown in the subsequent part of the text. Finally, Equation (3d) ensures that the number of allocated stocks does not exceed the volume of the manufactured output.

Let us discuss a sample procedure for finding a feasible value of X^*. Since the size of X would be very large in practical applications, we propose a two-step procedure.

In the first step, a model is constructed that minimises the deviation of production from the current demand assuming pessimistic availability of the critical parts:

$$\min \sum_{n \in N} \left(\sum_{d \in D} v_{dn} - y_n \right)^2, \tag{5}$$

subject to:

$$A\mathbf{y} \leq \mathbf{b} - \boldsymbol{\psi}, \tag{6a}$$

$$y_n \in \mathbb{N}_0. \tag{6b}$$

This model yields a pessimistic feasible value of production that matches demand V. We will denote that value as y^*. When the pessimistic value of production $\mathbf{y}^*$ is known, feasible allocation X^* can be calculated so that for each $n \in N$, the percentage deviation from the reported demand is minimised:

$$\min \sum_{d\in D; v_{dn}>0} \left(\frac{v_{dn}-x_{dn}}{v_{dn}}\right)^2, \tag{7}$$

subject to:

$$x_{dn} \le v_{dn}, \tag{8a}$$

$$\sum_{d\in D} x_{dn} \le y_n^*, \tag{8b}$$

$$x_{dn} \in \mathbb{N}_0 \ \forall d \in D, n \in N. \tag{8c}$$

Table 1. Notation summary

Input variables	
$n \in N$	product type, where N is the set of considered products
$d \in D$	buyer, where D is the set of buyers
$k \in K$	part type (critical component) required to manufacture a given type of product, where K is the set of part types
$b_k \in \mathbb{N}_0$	optimistic-assumption availability of critical components k
$\xi_k \in \mathbb{N}_0$	unknown perturbation to the availability of parts; the perturbation vector is denoted as $\boldsymbol{\xi} = [\xi_k]$
$\psi_k \in \mathbb{N}_0$	maximum possible perturbation of the availability of components $\psi_k \ge 0$; maximum perturbation vector is denoted as $\boldsymbol{\psi} = [\psi_k]$
$\Gamma \in \mathbb{N}_0$	maximum L^1 norm of the possible perturbations $\lVert\boldsymbol{\xi}\rVert_1 \le \Gamma$
$a_{kn} \in \mathbb{N}_0$	amount of critical parts k required to manufacture one unit of product n; the technology matrix is represented by $A = [a_{kn}]$
$s_{ij} \in \{0,1\}$	product substitution equivalent, where $s_{ij} = 1$ means that product i can be replaced by product j, where $i, j \in N$, and $s_{ij} = 0$ means that no replacement is possible, and the substitution matrix is represented by $S = [s_{ij}]$
$v_{dn} \in \mathbb{N}_0$	buyer demand d for product n; the demand matrix is represented by $V = [v_{dn}]$
p_{dn}	price acquired from selling product n to buyer d (prices vary across buyers due to different contract terms)
c_n	unit costs of manufacturing one unit of product n, in vector notation denoted as $\mathbf{c} = [c_n]$
x_{dn}^*	pessimistic level of the allocation of products; it can be represented by matrix $X^* = [x_{dn}^*]$
$\gamma \in [0,1]$	substitution rigidity; $\gamma = 0$ means that all customer orders can be replaced with substitutes, $\gamma = 1$ means that the level of substitutes will be minimised, and at least X^* will be fulfilled
$\rho(X)$	profit from production allocation X; optimistic and pessimistic profits are denoted as ρ^+ and ρ^-, respectively
Optimisation model variables	
y_n	production level of item n; it can be represented by vector $\boldsymbol{y} = [y_n]$
x_{dn}	number of goods of type n allocated to buyer d; it can be represented by matrix $X = [x_{dn}]$

Source: author's work.

Goal Function (7) involves minimising the percentage deviations of supply and demand. Note that for each $n \in N$, the exact solution to (7) can be easily achieved in three steps: (1) the proportional scaling of v_{dn} values in such a way that their sum is equal to y_n, (2) rounding those values down to the nearest integer and, finally, (3) redistributing the remaining product units starting from customers with the smallest orders. In practice, the decision-maker might decide to use substitution rigidity parameter γ to downscale the value of X^* and, as a result, offer more aggressively substitutes to their customer base instead of the ordered products, thereby generating a greater profit from the unbalanced market situation.

The dependencies between the subsequent modelling steps are presented in Figure 1. Firstly, we calculate the pessimistic production volume by solving the model presented in Equation 5. Secondly, we calculate the pessimistic amount of goods that will be made available to customers (please note that we are considering a post-pandemic economy with a scarcity of goods). It is important to remember that this solution will be feasible regardless of the observed perturbation value $\boldsymbol{\xi}$. Finally, since there is no more stable production capacity matching the demand, the company will manufacture product substitutes that will also be accepted by the market. This process is controlled by substitution rigidity parameter γ– its actual value will depend on the business objectives of the company.

It is assumed that since the demand on the market is high, customers will purchase substitute goods as long as Equation (3b) holds. Please note that due to uncertainty $\mathbf{U}$ (see Equation (1)), the decision-maker faces risk related to cash flow management and needs to adjust the production plan accordingly.

Given uncertainty set $\mathbf{U}$, the pessimistic value of profits ρ^- can be calculated by solving the following model:

$$\rho^- = \min_{\xi \in U}\left\{max \sum_{n \in N, d \in D} x_{dn} p_{dn} - \mathbf{c}^T \mathbf{y}; \quad subject\ to\ constr. 3a\text{–}\ 3f\right\}. \tag{9}$$

On the other hand, the optimistic value of profits ρ^+ can be computed by means of solving Equation (2), assuming that $\boldsymbol{\xi} = \mathbf{0}$. Bertsimas et al. (2016) point out that there are several approaches to reformulating a robust MILP optimisation model into a set of MILP models, but they all assume that perturbations are defined individually for each constraint (see e.g. Ben-Tal et al., 2009 or Li et al., 2011). Since the model presented in Equation (9) is the mixed integer programming, and the uncertainty set is defined across all constraints, it can only be solved through iterating solutions over the entire set $\mathbf{U}$.

For larger sizes of uncertainty set **U**, iterating over all of its values is prohibitively computationally expensive. However, nearly optimal solution can be found by using a cutting plane heuristic similar to the one proposed by Bertsimas et al. (2016).

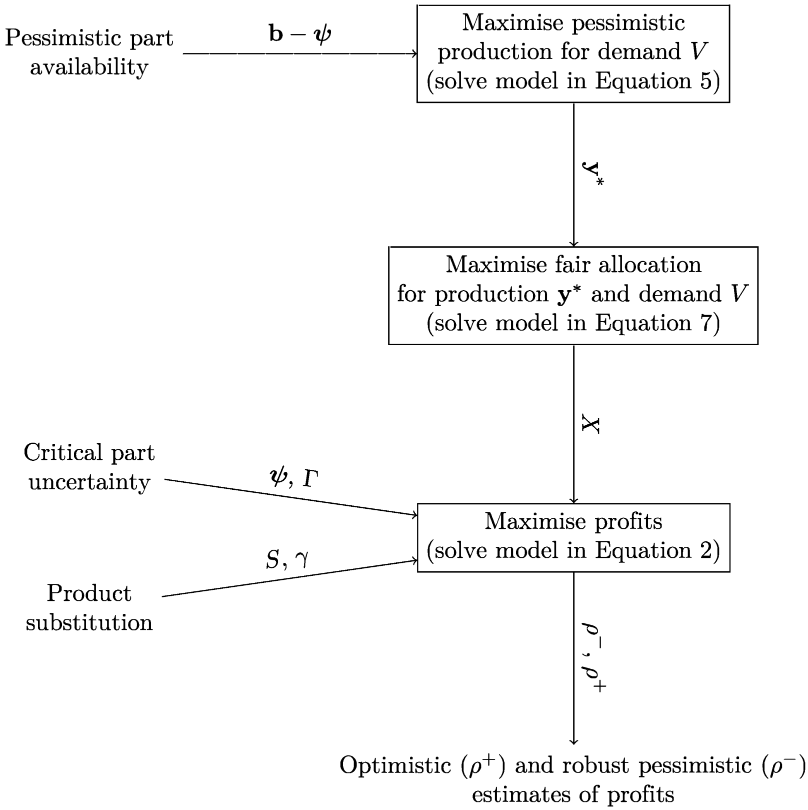
In this paper, the following algorithm for estimating pessimistic profit value ρ^- was developed:

$\Gamma^* = \Gamma$
$\boldsymbol{\xi} = \mathbf{0}$
$\rho^* = \mathbf{0}$
while $\Gamma^* > 0$ **do**
    $\rho_0^* = \rho^*$
    for each $k \in K$ **do**
        $\boldsymbol{\xi}' = \boldsymbol{\xi}$
        $\xi_k' = \xi_k + \min\{\psi_k - \xi_k, \Gamma^*\}$ ▷ Try cutting to the furthest possible extent
        $\boldsymbol{X}, \boldsymbol{y}$ = solve Equation 2 with fixed perturbation $\boldsymbol{\xi} = \boldsymbol{\xi}'$
        $\rho = \sum_{n \in N, d \in D} x_{dn} p_{dn} - \boldsymbol{c}^T \boldsymbol{y}$
        if $\rho < \rho^*$ **then**
            $\rho^* = \rho$
            $k^* = k$
            $\xi^* = (\sum_{n \in N} a_{kn} y_n) - b_k$ ▷ Reduce cut to the amount sufficient to obtain ρ
        end
    end
    $\Gamma^* = \Gamma^* - (\xi^* - \xi_{k^*})$
    $\xi_{k^*} = \xi^*$
    if $\rho^* - \rho_0^* < \epsilon$ **then**
        break ▷ No significant improvement found
    end
end
return ρ^*

Source: author's work.

The idea behind the heuristic presented in the above algorithm is to sequentially select the constraint that leads to the highest reduction of the cost function when under perturbation. At a given step of the algorithm, we are sequentially considering each constraint $k \in K$. For a given constraint k, we assign its maximum perturbation level so that it still satisfies the boxed-polyhedral uncertainty inequality presented in Equation (1) and solves the optimisation problem denoted by Equation (2). Finally, we choose a constraint whose perturbation leads to the highest reduction of the goal function presented in Equation (2). Once the new value is calculated, we set the perturbation level to eliminate the unnecessary slack. The algorithm stops when no significant improvement is found (the minimal improvement value is presented as $\epsilon > 0$ in the algorithm).

Figure 1. Dependencies between the proposed optimisation models



Source: author's work.

3. Numerical experiments

The goal of this section is twofold. Firstly, we will show the numerical accuracy of the algorithm from Section 2.2. Secondly, the sensitivity of the model's parameters will be demonstrated on sample input data.

The optimisation model presented in the previous section has been implemented in the Julia programming language. We conducted numerical experiments using Julia JuMP (Dunning et al., 2017; Legat et al., 2022).

The parameters for the numerical experiments are presented in Table 2. Please note that $\sim \{\dots\}$ indicates that a value is uniformly drawn from the given set, $\sim N(\mu, \sigma)$ means that a value is chosen from the normal distribution with mean μ and standard deviation σ, and $\sim U(a, b)$ denotes a number drawn from the uniform distribution. Random values are also used in the model in its equations – in this case they are shown in parentheses, e.g. $\max\left(0, (\sim N(4, 7))\right)$ means a censored normal distribution where the negative values of the left tail are replaced with zeros. Finally, please note that whenever a continuous distribution is used, all the values are rounded to the nearest integer.

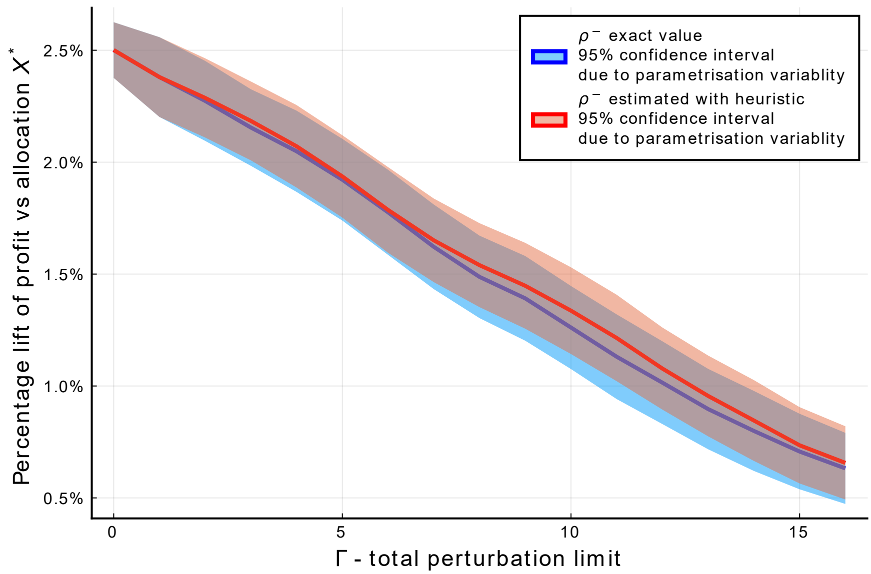
Table 2. Parameters used for numerical experiments

Parameter	Symbol	Value
Number of item types	$\|N\|$	15
Number of distributors	$\|D\|$	15
Number of critical part types	$\|K\|$	6
Substitution matrix	S	$s_{ii} = 1$ and $\forall_{i \neq j} P(s_{ij} = 1) = 1/3$
Technology matrix	a_{kn}	$\sim \{0,1\}$
Prices	p_{dn}	prices are generated for each value of $n \sim \{101 + (n-1) * \|D\|, \ldots, 100 + n * \|D\|\}$
Cost	c_n	$c_n = p_{1,n}/2$
Demand	v_{dn}	$\max\left(0, \left(\sim N(4,7)\right)\right)$
Part availability	b_k	$\sum_{n \in N}\left(a_{kn} \sum_{d \in D} v_{dn} - \left(\sim U(1,20)\right)\right)$
Maximum perturbation	ψ_k	$\sim \{4, 5, 6\}$
Total perturbation limit	Γ	1–16 (numerical accuracy) 5–40 (model properties)

Source: author's work.

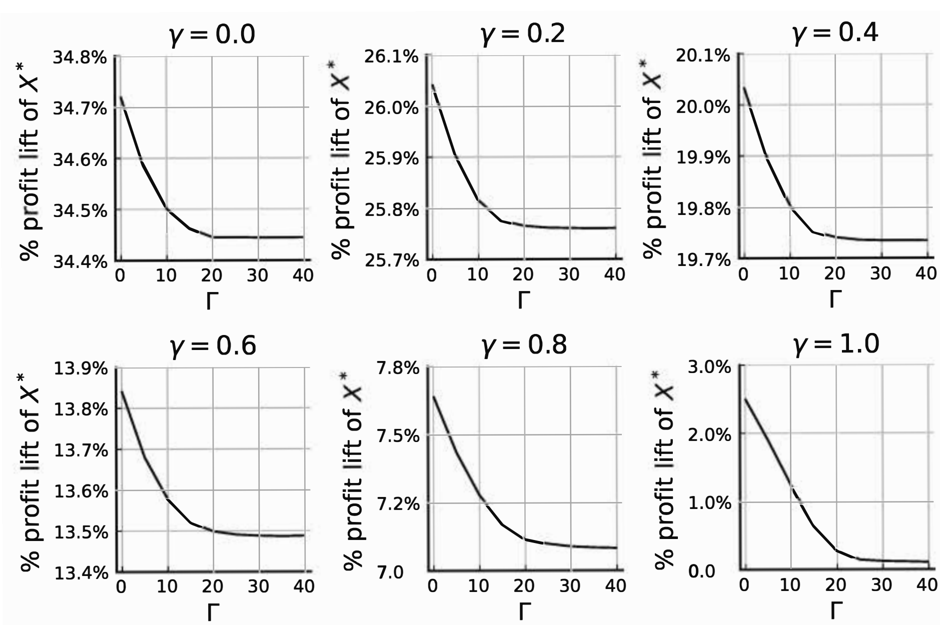
We start by evaluating the quality of the cutting plane heuristic proposed in the algorithm from Section 2.2. Following the parametrisations presented in Table 2, 30 different randomised scenarios were constructed, including demand structure, prices, substitution matrices and critical part availability. For each of those scenarios, the pessimistic value of profit ρ^- was evaluated in two ways. Firstly, ρ^- was fully enumerating all values $\boldsymbol{\xi} \in \mathbf{U}$ and solving a separate optimisation model, hence obtaining the exact solution to the robust optimisation problem presented in Equation (9). Secondly, the same ρ^- value was estimated with the algorithm. The results are presented in Figure 2, and scaled against the profit that can be obtained in the pessimistic scenario without substitution (X^*). The profit lift is calculated as $(\rho(X)/\rho(X^*) - 1) * 100\%$. It becomes evident that the heuristic yields a slightly larger estimate of profit ρ^- compared to the actual exact solution; however, this difference is marginal considering the influence of the other model parameters – any change of Γ has a much more significant impact on the results.

Figure 2. Performance of the algorithm developed in the paper vs. the exact solution



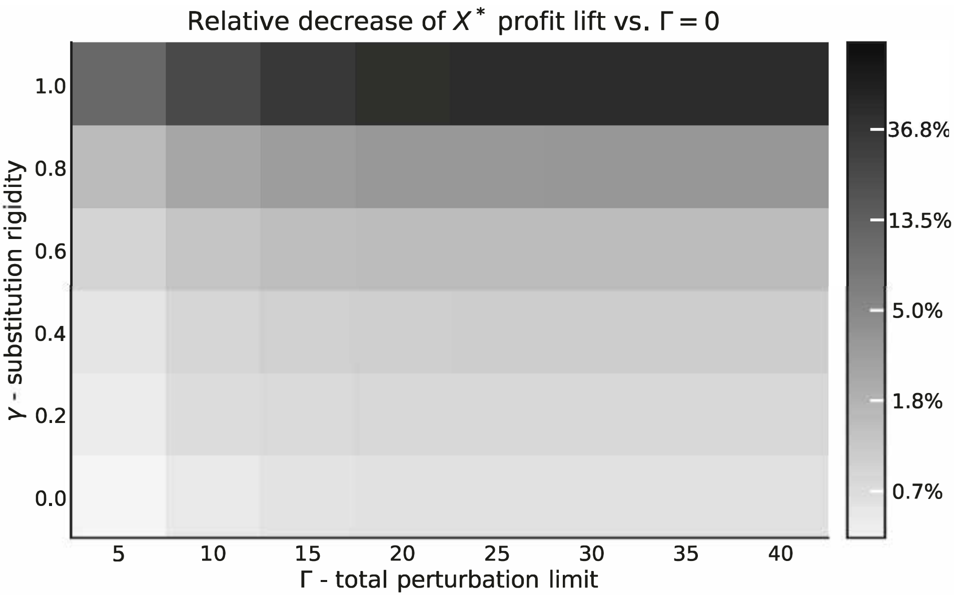
Source: author's calculations.

Figure 3. Additional pessimistic estimate of profits ρ^- acquired due to substitution at various total perturbation limits Γ and substitution rigidity levels γ



Source: author's calculations.

Figure 4. Impact on the overall profit of substitution rigidity and the total perturbation limit



Source: author's calculations.

Figure 3 shows how substitution rigidity γ jointly with total perturbation limit Γ influence the model outcomes. The profit that can be obtained at rigid pessimistic solution X^* is used as a benchmark. Similarly to the previous plot, the profit lift value is calculated as $(\rho(X)/\rho(X^*) - 1) * 100\%$. Figure 3 demonstrates that regardless of the substitution rigidity level γ, the pessimistic estimate of profits ρ^- drops with the increase of the total perturbation limit Γ, which is the expected outcome. However, it is worth noting that the marginal drops decrease as the values of Γ increase.

Figure 4 shows to what extent the presented model is sensitive to maximum perturbation level Γ at various levels of product substitution rigidity γ. Again, the profit lift $(\rho(X)/\rho(X^*) - 1)$ is used as the benchmark value (note the log scale of the colour bar). However, in order to ensure comparability, the results for each rigidity level γ have been scaled using the optimistic profit lift value, i.e. the profit lift without perturbation $(\Gamma = 0)$. It can be seen that when there is a high product substitution rigidity $(\gamma = 0)$, the model is very sensitive to the perturbation limit. On the other hand, when even small level of substitution is possible, the business importance of perturbation limit Γ quickly diminishes. This means that if customers are even slightly inclined to buy product substitutes on a market with shortages, then it immediately has a considerable impact on the number of goods that can be manufactured. Hence, even a small substitution flexibility yields a significant increase in profits.

The numerical experiments show that the heuristic proposed in the algorithm ensures a sufficient level of accuracy to apply the model in supporting a real production system. The model proposed in the paper applied using real-life data allows a more precise measurement of the effects that product substitutes have on the actual operational efficiency of the company. It also shows that ensuring even small substitution elasticity can have a significant influence on the financial results of a manufacturing enterprise.

4. Conclusions

In this paper, a robust optimisation model was presented which maximises the profits of a manufacturing company located in the middle of a supply chain. This kind of a company struggles with uncertain supplies of sub-components and experiences market disturbances, and therefore is willing to offer its customers substitutes instead of the originally requested products. The model developed in the paper was implemented in the Julia programming language and tested in a series of numerical experiments.

The main outcomes are as follows: (1) an integrated model for profit maximisation in a production company facing uncertain supplies of critical components at various levels of product substitution rigidity, (2) a heuristic that makes it possible to efficiently solve the presented problem at scale, (3) a set of business guidelines on how the product substitution rigidity and component availability perturbations affect the final financial situation of a company, and (4) managerial insights for decision-making in post-pandemic markets. The proposed results and methodology can be immediately applied to companies operating on today's markets prone to unbalanced demand and sub-component shortages.

The research presented in the paper can be expanded on in many ways. One of them is multi-period planning for resources, i.e. adding another dimension of weeks or months to the production plan. This would significantly increase the computational conditionality of the model. Another possible direction of the future research could involve the construction of an agent-based model (e.g. see Tesfatsion, 2003) of an entire shortage-driven economy. Such a model would take into consideration several manufacturers in the logistic chain, so that the output of one manufacturer would be the input for another.

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